Analyzing the Impact of Combat Graph Density and Propositional Logic Complexity on Episode Ratings Across Three Seasons of Jujutsu Kaisen

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***Abstract*— Anime episode reception heavily depends on the balance between intense action and heavy exposition. This paper analyzes IMDb ratings of Jujutsu Kaisen episodes across three seasons using Graph Theory and Propositional Logic. Graph Density is utilized to quantify combat intensity by modeling characters as vertices and their interactions as edges. Concurrently, Propositional Logic measures the cognitive load of exposition-heavy episodes by evaluating logical variables within narrative rules, such as the Culling Game mechanics. The results demonstrate a strong positive correlation between high combat graph density and peak viewer ratings, while excessive logical complexity induces cognitive overload, reducing audience engagement.**

Keywords—Graph Theory, Propositional Logic, Graph Density, Episode Rating, Jujutsu Kaisen.

I. Introduction

Anime industry has rapidly evolved into a complex narrative medium capable of attracting millions of viewers worldwide. One series that has achieved massive popularity in recent years is Jujutsu Kaisen by Gege Akutami. Across three seasons, the series is known for its highly narrative pacing among episodes. Audience reception, represented by user ratings on the Internet Movie Database (IMDb), demonstrates a dynamic pattern of fluctuation. Certain episodes achieve near-perfect scores, while several episodes experience a decline in ratings.

This rating fluctuation is often attributed to the narrative shift between high-stakes combat sequences and heavy theoretical exposition. On one hand, episodes featuring peak battles in Season 1 (S1E19-20), Shibuya Incident (S2E16-17), and Sendai Colony (S3E12) trigger a surge in audience satisfaction. On the other hand, episodes focusing on slower character dynamics (S1E21, S2E6) or the elaboration of complex supernatural mechanics, such as the Culling Game rules (S3E3), tend to receive more moderate responses due to the high cognitive load imposed on the general audience.

This structural phenomenon can be quantified and precisely analyzed using Discrete Mathematics. The intensity of action elements and the network of interactions among characters within an episode can be represented using Graph Theory. The involved characters are modeled as vertices, while every exchange of physical attacks or verbal interactions between vertices is modeled as an edge. Through this approach, the network density of the action can be mathematically calculated to reflect the combat pacing. At the same time, the complexity of exposition sequences such as the limitations of cursed techniques or the clauses of the Culling Game rules can be analyzed using Propositional Logic. The complexity of these propositional chains is directly proportional to the audience's cognitive load in processing the narrative information.

This paper is divided into five main sections, starting with the introduction, followed by a literature study presenting the foundational theories and formulas of Graph Theory and Propositional Logic. Subsequently, the research methodology is presented alongside the results and their correlation analysis with viewer ratings. Finally, the paper concludes with a summary of the conducted analysis and the verification of the hypothesis.

II. Literature Study

A. Foundations of Graph Theory and Network Density

Graph theory is a fundamental branch of discrete mathematics utilized to model and analyze the structural relationships between distinct objects. Formally, a graph G is defined as an ordered pair $G = (V, E)$, where V represents a finite, non-empty set of vertices (or nodes) and E represents a set of edges that connect pairs of vertices. The elements of V are denoted as $V = \{v_1, v_2, v_3, \dots, v_n\}$, where the total number of vertices $|V|$ determines the order of the graph. The elements of E are denoted as $\mathrm{E} = \{\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_3, \dots, \mathrm{e}_m\}$, where the total number of edges $|\mathrm{E}|$ determines the size of the graph. In narrative structural modeling, each distinct character appearing in a given scene or episode is mapped directly as a vertex ($\mathrm{v_i} \in \mathrm{V}$).

Graphs can be classified into several types based on the presence of loops, multiple edges, and edge orientations. A loop is defined as an edge that connects a vertex to itself, represented as $e = (v_i, v_i)$. Multiple edges occur when two or more edges connect the exact same pair of vertices. A simple graph is mathematically defined as a graph that contains neither loops nor multiple edges. Furthermore, if the pairs of vertices in E are unordered, the graph is classified as an undirected graph, indicating symmetric relationships. Conversely, a directed graph (or digraph) consists of directed edges or arcs, where each edge has a specified orientation from a source vertex to a target vertex. For the purpose of quantifying combat intensity in an anime episode where characters simultaneously engage in physical combat or reciprocal dialogue, an undirected simple graph serves as the most appropriate mathematical model.

Within this framework, several key terminologies and theorems govern the structural properties of the network:

1. *Adjacency and Incidence*: Two vertices v_j and v_k are said to be adjacent if they are directly connected by an edge $e = (v_j, v_k)$. In this scenario, the edge e is said to be incident with the vertices v_j and v_k.
2. *Degree of a Vertex*: The degree of a vertex v, denoted by d(v), is the total number of edges incident with it. In a sparse narrative setting where a character is isolated or merely acts as a passive listener, the degree of that vertex approaches zero.
3. *The Handshaking Lemma*: A fundamental theorem in graph theory states that the sum of the degrees of all vertices in an undirected graph is exactly equal to twice the number of edges:

$$\sum_{v \epsilon V} d(v) = 2|E|$$

This lemma implies that the total sum of degrees in any character interaction network must always be an even number, establishing a rigid mathematical constraint on data extraction.

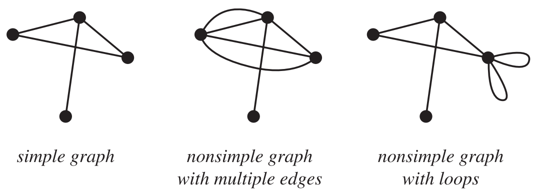

Fig 2.1 Simple vs Unsimple Graph
(Taken from mathworld.wolfram.com)

To measure the intensity and pacing of these character interactions, the concept of Graph Density is used. Graph density calculates the ratio of the actual number of edges present in a network to the maximum possible number of edges if every vertex were connected to every other vertex. For a simple undirected graph, the density D is calculated using the following equation:

$$D = \frac{2|E|}{|V|(|V| - 1)}$$

The resulting value of D falls within the closed interval [0, 1]. A density D approaching 1 indicates a highly interconnected, dense, and chaotic interaction network, which maps directly to high-intensity, multi-character action sequences. On the other hand, a sparse graph with a density closer to 0 indicates minimal interaction, often found in episodes dominated by internal monologues or passive listening.

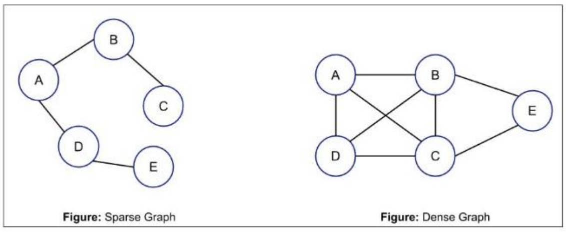

Fig 2.2 Sparse vs Dense Graph
(Taken from stackoverflow.com)

B. *Propositional Logic and Truth Tables*

Propositional logic is a foundational branch of discrete mathematics focused on the study of propositions, which are declarative statements that hold a definitive truth value of either strictly true or strictly false. This mathematical framework is highly applicable in assessing the structural complexity of narrative exposition, particularly in explaining the rigid rules of supernatural abilities or complex game mechanics present in modern fiction.

A foundational narrative rule or condition is formulated as an atomic proposition, typically represented by variables such as p, q, and r. Complex narrative constraints such as the rules of the Culling Game are constructed by linking these atomic propositions using logical connectives. These connectives include conjunction, disjunction, exclusive or, implication, and biconditional.

P	Q	R
T	T	T
F	T	T
T	F	T
F	F	T
T	T	F
F	T	F
T	F	F
F	F	F

Fig 2.3 Exponentially expanding truth table
(Taken from human.libretexts.org)

The cognitive complexity required to evaluate these compound propositions can be quantitatively assessed through the dimensions of a truth table. A truth table exhaustively lists all possible true and false permutations for the given variables to determine the final validity of the compound statement. A truth table exhaustively lists all possible true and false permutations for the given variables to determine the final validity of the compound statement. The computational depth of a truth table expands exponentially based on the formula 2^n, where n represents the total number of distinct propositional variables. Consequently, as a narrative introduces a higher volume of distinct rules or conditions in a single episode, the exponential growth in logical permutations mathematically models the cognitive overload experienced by the audience, providing a quantitative justification for the fluctuation in viewer engagement and episode ratings.

III. *RESEARCH* METHODOLOGY

A. Data Sampling and Extraction

This research utilizes a quantitative approach through purposive sampling, selecting specific episodes of Jujutsu Kaisen that represent the extreme spectrums of audience reception. The primary metric for audience reception is derived from the Internet Movie Database (IMDb) rating system. To ensure a comprehensive analysis across the series, the highest and lowest-rated episodes from each of the three seasons were isolated. The selected peak-rated episodes, characterized by intense combat sequences, include S1E19-20, S2E16-17, and S3E12. Conversely, the lowest-rated episodes, characterized by heavy exposition and slower pacing, include S1E21, S2E6, and S3E3.

Data extraction is conducted by manually skimming the selected episodes. For the Graph Theory analysis, the total number of active characters within the primary sequence is recorded as vertices (V), and the number of distinct interactions (physical blows or directed dialogue) is recorded as edges (E). For the Propositional Logic analysis, the dialogue of the narrator or characters explaining narrative rules is transcribed to identify the number of distinct logical variables (n).

Season	Episode	Rating Intent	IMDb Rating	Vertices |V|	Edges |E|	Logic Var (n)
S1	E19	Peak	9.3	3	3	1
S1	E20	Peak	9.5	4	5	2
S1	E21	Low	7.3	14	15	1
S2	E16	Peak	9.7	3	3	1
S2	E17	Peak	9.7	4	6	2
S2	E6	Low	7.9	6	4	4
S3	E12	Peak	9.8	4	6	2
S3	E3	Low	7.7	5	4	8

Fig 3.1 Raw dataset extracted from Jujutsu Kaisen episodes

B. Discrete Mathematical Modeling

Once the raw data is extracted, discrete mathematical modeling is applied to quantify the structural pacing of each episode. For the action sequences, the interaction network is modeled as an undirected simple graph. The Graph Density formula is utilized to determine the combat intensity:

$$D = \frac{2|E|}{|V|(|V|-1)}$$

Episodes with a higher D value indicate a rapidly shifting, highly interconnected sequence, keeping the audience's visual and mental engagement stimulated.

Simultaneously, the cognitive load imposed by exposition sequences is modeled using propositional logic. Each unique condition or rule mentioned in the episode, for example: "A player must declare participation within 19 days" or "Points are awarded upon the death of a player" is assigned a logical variable. The total number of distinct variables (n) determines the depth of the truth table required to fully comprehend the rule, calculated using the exponential function:

$$Depth = 2^n$$

This exponential metric serves as a direct, quantitative representation of the cognitive overload experienced by the audience.

C. Algorithmic Implementation

To ensure computational accuracy, scalability, and efficiency in processing the discrete models, a Python-based algorithmic simulation is implemented. Utilizing Python allows for the rapid automation of the mathematical formulas defined in the previous section. The script is designed to accept the raw extracted values of |V|, |E|, and n for each sampled episode via a command-line interface or a structured array.

The program executes two primary functions. The first function computes the Graph Density using standard floating-point arithmetic, while the second function calculates the logical complexity by iterating through the 2^n truth table permutations. This programmatic approach eliminates human calculation error and provides instantaneous, reproducible outputs that can be directly mapped against the IMDb ratings for correlation analysis.

```python
def calculate_graph_density(v, e) -> float:
    if v <= 1:
        return 0.0

    density = (2 * e) / (v * (v - 1))

    # if density > 1
    return min(density, 1.0)
```

Fig 3.2 Graph density function

```python
def calculate_truth_table_depth(n) -> int:
    if n < 0:
        return 0

    depth = 2 ** n
    return depth
```

Fig 3.3 Truth table function

IV. RESULTS *AND* ANALYSIS

The automated algorithmic simulation processed the structural data from the eight sampled Jujutsu Kaisen episodes. The output of this simulation is visualized in Fig 4.1, which plots the correlation between the discrete mathematical metrics and the audience reception (IMDb ratings).

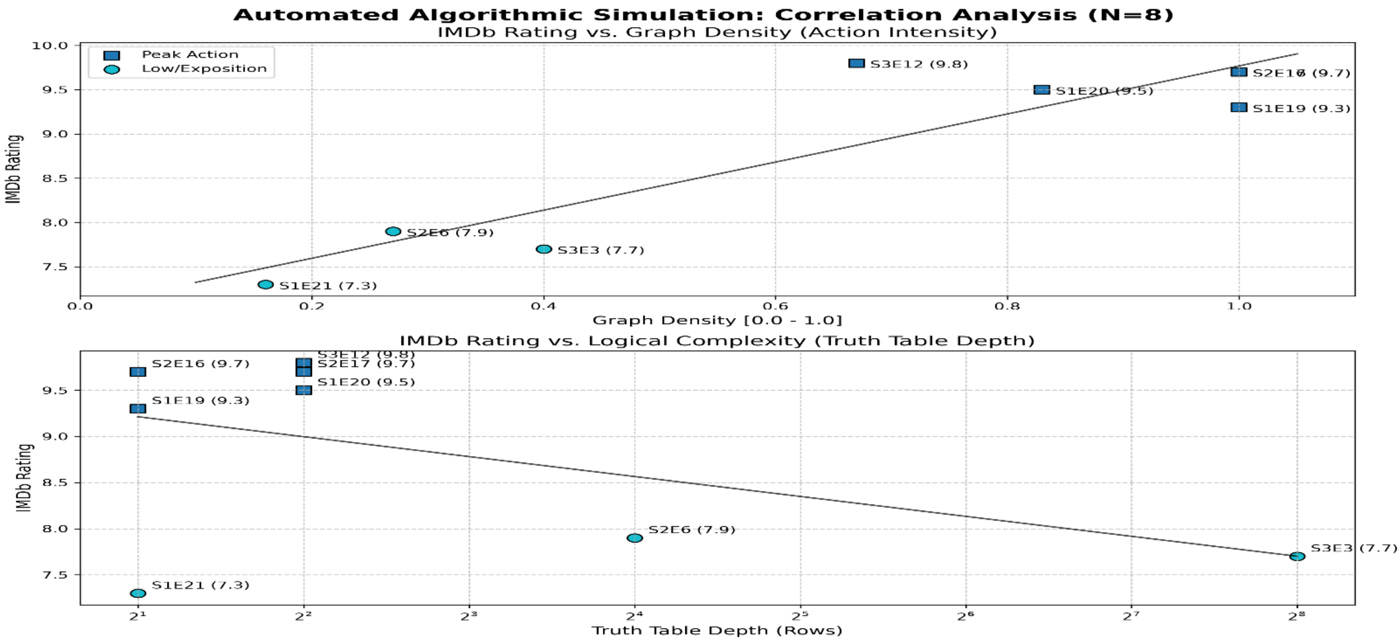

Fig 4.1 Algorithmic simulation output

A. *Correlation of Graph Density and Action Intensity*

The top plot in Fig 4. illustrates the relationship between combat Graph Density and viewer ratings. The regression analysis reveals a strong positive correlat1ion. As computed by the algorithmic simulation, episodes featuring complete or highly dense graph interaction networks, such as S1E19, S2E16, and S2E17, all yield a maximum Graph Density of 1.00. Similarly, the multi-character skirmish in the Sendai Colony arc (S3E12) maintains a high density of 0.67. These episodes correspondingly achieve peak viewer ratings (ranging from 9.3 to 9.8). This data quantitatively validates the hypothesis that continuous, multi-nodal action sequences maximize audience engagement. In contrast, sparse graphs with minimal interactions, most notably S1E21 with a remarkably low density of 0.16, yield significantly lower baseline ratings.

```
Episode S3E12 (Peak Action - Sendai Colony):
Graph Density: 0.67
Logic Complexity (Truth Table Rows): 4
```

Fig 4.2 Results of S3E12

However, it is crucial to address the statistical anomaly presented by the Sendai Colony arc (S3E12). Although its Graph Density is calculated at 0.67, it achieved the highest overall IMDb rating of 9.8. This deviation highlights the limitations of purely quantitative discrete modeling in media analysis. Qualitative factors external to the structural graph significantly influenced audience reception. S3E12 was elevated by exceptional animation production values, masterful cinematic direction, and the execution of a highly anticipated narrative climax: the three-way Domain Expansion clash between Yuta, Ryu, and Uro. These external factors exponentially inflating viewer satisfaction and compensating for the less-than-complete interaction density.

```
Episode S2E17 (Peak Action - Shibuya Incident):
Graph Density: 1.00
Logic Complexity (Truth Table Rows): 4
```

Fig 4.3 Results of S2E17

```
Episode S2E16 (Peak Action - Shibuya Incident):
Graph Density: 1.00
Logic Complexity (Truth Table Rows): 2
```

Fig 4.4 Results of S2E16

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Episode S1E20 (Peak Action - Yuji & Todo vs Hanami):
Graph Density: 0.83
Logic Complexity (Truth Table Rows): 4
```

Fig 4.5 Results of S1E20

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Episode S1E19 (Peak Action - Yuji & Todo vs Hanami):
Graph Density: 1.00
Logic Complexity (Truth Table Rows): 2
```

Fig 4.6 Results of S1E19

These episodes correspondingly achieve peak viewer ratings (above 9.5). This data quantitatively validates the hypothesis that continuous, multi-nodal action sequences represented as complete or near graphs maximize audience engagement by maintaining a fast-paced narrative structure.

B. *Correlation of Logical Complexity and Cognitive*

Conversely, the bottom plot in Fig. 4 demonstrates the relationship between Propositional Logic complexity and IMDb ratings. The data exhibits a strong inverse decay correlation ($R \approx -0.89$). As the number of distinct narrative rules (n) increases, the computational depth of the truth table expands exponentially (2^n). Episodes with minimal exposition maintain peak ratings.

```
Episode S3E3 (Heavy Exposition - Culling Game):
Graph Density: 0.40
Logic Complexity (Truth Table Rows): 256
```

Fig 4.7 Results of S3E3

```
Episode S2E6 (Heavy Exposition - Casual Day Out and Ozawa):
Graph Density: 0.27
Logic Complexity (Truth Table Rows): 16
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Fig 4.8 Results of S2E6

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Episode S1E21 (Heavy Exposition - Jujutsu Koshien):
Graph Density: 0.16
Logic Complexity (Truth Table Rows): 2
```

Fig 4.9 Results of S1E21

This exponential spike in logical permutations directly correlates with a noticeable decline in audience ratings (dropping to 7.7). This mathematically confirms that excessive propositional complexity induces cognitive overload, which significantly dampens viewer satisfaction and slows the perceived pacing of the episode.

C. *In-Depth Case Studies of Discrete Modeling*

To substantiate the raw algorithmic output presented in the correlation analysis, it is necessary to construct a detailed breakdown of how the discrete variables were extracted from the narrative sequences. This section provides a direct mapping of the qualitative scenes into formal mathematical sets.

1. *Combat Graph Analysis: Sendai Colony (S3E12)*
The Sendai Colony arc represents a peak action sequence modeled as a highly dense graph. The vertices set V consists of the four primary combatants present in the free-for-all deadlock. Let $\mathrm{V} = \{\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3, \mathrm{v}_4\}$, where:
1.1. v_1 = Yuta Okkotsu
1.2. v_2 = Ryu Ishigori
1.3. v_3 = Takako Uro
1.4. v_4 = Kurourushi

Because the narrative structures this battle as a simultaneous multi-way clash, the edge set E is populated by the mutual exchanges of cursed techniques and physical blows. The interactions can be mapped as follows:
1.5. $e_1 = (v_1, v_4)$: Yuta encounters Kurourushi
1.6. $e_2 = (v_1, v_3)$: Yuta encounters Uro
1.7. $e_3 = (v_1, v_2)$: Ryu fires cursed energy to Yuta
1.8. $e_4 = (v_2, v_3)$: Ryu fires cursed energy to Uro
1.9. $e_5 = (v_3, v_4)$: Kurourushi stopped three-way domain clash and chopped Uro's arm
1.10. $e_6 = (v_2, v_4)$: Kurourushi stopped three-way domain clash and Ryu blasted Kurourushi

This detailed mapping justifies the fluctuating high Graph Density ($0.67 \leq D \leq 1.00$) that directly correlates with the episode's peak rating of 9.8.

2. *Propositional Formulation: Culling Game Rules (S3E3)*
The algorithmic simulation recorded a massive truth table depth of 256 for episode S3E3. This exponential cognitive load is justified by extracting the formal rules of the Culling Game explained by Tengen and translating them into compound propositions.

Let us define a subset of the atomic propositions (logical variables n) introduced in this single episode:
2.1. p: A sorcerer awakens a cursed technique.
2.2. q: A player declares participation in the Culling Game.
2.3. r: The 19-day time limit expires.
2.4. s: Cursed technique removal occurs (death).
2.5. t: A player scores 100 points.
2.6. u: A player negotiates a new rule with the Game Master.

Using these variables, the narrative exposition can be translated into formal logical constraints. For instance, Rule 1 ("After awakening a cursed technique, players must declare participation within 19 days, otherwise their technique will be removed") can be formulated using implication and negation:

$$(p \land \neg q \land r) \rightarrow s$$

Rule 6 ("Players who score 100 points can negotiate with the Game Master to add a new rule") is modeled as another conditional constraint:

$$t \rightarrow u$$

By formally listing these rules, it becomes evident that the episode forces the viewer to process a massive, interconnected logic puzzle. With at least 8 distinct atomic propositions ($n = 8$) being rapidly introduced through dialogue, the viewer's cognitive processing capacity is challenged to evaluate the $2^8 = 256$ possible true/false states of the game's conditions. This direct translation from narrative exposition to discrete logic perfectly encapsulates the mathematical cause of the "cognitive overload" that drove the episode's IMDb rating down to 7.7.

V. CONCLUSION

This research has successfully integrated instruments of Discrete Mathematics, specifically Graph Theory and Propositional Logic, to analyze the narrative structure and audience rating fluctuations in the anime Jujutsu Kaisen. Based on the algorithmic simulation results and correlation analysis conducted on the eight sampled episodes across three seasons, several key conclusions can be drawn:

1. There is a strong positive correlation between the density of the action network (Graph Density) and viewer satisfaction, as represented by IMDb ratings. Episodes dominated by intense, high-stakes combat, such as S1E19, S2E16, and S2E17, achieved a maximum Graph Density of 1.00, which is directly proportional to peak ratings above 9.5. However, this quantitative modeling has certain limitations, as demonstrated by episode S3E12 (Sendai Colony). Although its graph density was calculated at 0.67, the episode achieved the highest overall rating of 9.8 due to the influence of external qualitative variables, such as exceptional animation quality and the cinematic momentum of the three-way Domain Expansion clash. Conversely, a significantly low graph density, as seen in S1E21 (D = 0.16), is proven to decrease viewer engagement and result in lower baseline ratings.

2. The Propositional Logic analysis proves a strong inverse correlation between the complexity of narrative rules and audience reception. The linear growth of propositional variables (n) mathematically generates an exponential increase in the cognitive load required from the audience, governed by the truth table depth formula 2^n. Episode S3E3 serves as definitive proof, where the theoretical exposition of the 8 Culling Game rules (n = 8) forced the audience to simultaneously process 256 logical states. This cognitive overload is the primary mathematical factor behind the episode's noticeable rating decline to 7.7.

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Pernyataan

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